Draft SPOTS Standard Part III (5)

CALIBRATION
AND ASSESSMENT
OF OPTICAL STRAIN
MEASUREMENTS

Good Practice Guide to Reflection Photoelasticity

December 2005

1. Scope

Photoelastic stress analysis is a technique of experimental stress analysis that utilises the temporary birefringence exhibited by transparent materials subjected to strain. When such materials are viewed using polarised light, fringes are observed that are related to the direction and magnitude of the strains present. The technique can be employed for stress analysis either directly for transparent objects; using transparent models which may be two- or three-dimensional; or by using a transparent polymer coating that is bonded to an opaque object and acts as a strain witness. A polariscope is used to observe the photoelastic fringes and quantitative analysis can be performed manually or using digital processing methods.

This is one of a pair of guides describing transmission and reflection photoelasticity in sufficient detail to allow experiments to be planned, simple tests to be executed and an initial level of data interpretation to be achieved.

2. Reference materials

ISO Norms

ISO IEC 17025:1999: "General requirements for the competence of testing and calibration laboratories"

ISO 10012:2003: "Measurement Management Systems - Requirements for measurement processes and measuring equipment"

ISO/TAG4/WG3, "Guide to the Expression of Uncertainty in Measurement" (GUM), 1995, identical with EN13005:1999: "Guide to the Expression of Uncertainty in Measurement"

ASTM Standards

E 2208-02 Standard Guide for Evaluating Non-Contacting Optical Strain Measurement Systems
C1426-99 Standard practices for verification of calibration of polarimeters. 1999.

Technical Notes[♣]

TN-704: “How to select photoelastic coatings”, Vishay Measurements Group, Inc., Raleigh NC., 1978.

TN-701: “Calibration of photoelastic coatings”, Vishay Measurements Group, Inc., Raleigh NC., 1977.

TN-706-1: “Corrections to photoelastic fringe-order measurements”, Vishay Measurements Group, Inc., Raleigh NC., 1992.

Bulletin S-116: “Photoelastic Materials and Coatings”, Vishay Measurements Group, Inc., Raleigh NC., 1978.

Bulletin IB-221: “Instructions for casting and contouring photoelastic sheets”, Vishay Measurements Group, Inc., Raleigh NC., 2001.

Other

VDI/VDE 2634 - Practical acceptance & verification methods for the evaluation of accuracy

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[♣] Extracts from these technical notes are reproduced with permission from Vishay Measurements Group Inc.

3. Symbols and abbreviations

Symbol	Definition	Units
C, C_{PS}, C_B, $C_{B\Delta}$	Correction factors	-
ΔD	Calibration beam deflection	m
E_c, E_s	Young's modulus of coating, specimen	Nm^{-2}
E_R	= (Ec /Es)	-
f	Fringe value of coating	m/m / fringe
K, K'	Strain Optic coefficient, corrected value	fringe
n	Fractional fringe order	-
N, N_i, N_f	Isochromatic fringe order, parasitic value, measured value	-
$N_0(\Delta T)$	Birefringence due to temperature change only	-
N_θ	Fringe order in oblique incidence	-
t_A	Adhesive thickness	m
t_B	Calibration beam thickness	m
$t_{c,s}$	Thickness of coating, specimen	m
t_R	= (tc /ts)	-
α_c, α_s	thermal expansion coefficients of coating, structure	C^{-1}
β	Rotation of analyser	rad
$\varepsilon_{1,2}$	Maximum, minimum principal strains	-
$\theta, \theta_{I}, \theta_f$	Isoclinic angle, parasitic value, measured value	degrees
λ	Wavelength of light	m
υ_c	Poisson's ratio of coating	-
$\sigma_{1,2}$	maximum, minimum principal stress parallel to the surface	Nm^{-2}

4. Terminology

Birefringent material – a material which is optically anisotropic, i.e. the refractive index varies with direction within the material.

Isochomatic fringe – fringes of constant colour observed when a birefringent material is placed in a polariscope. They are contours of constant maximum shear stress.

Isoclinics – black fringes produced at locations where the principal strain directions are coincident with the polarizing axes of a plane polariscope (one in which the quarter wave-plates have been removed or are optical inactive)

Polariscope – a device consisting of a light source and two pairs of polarisers and quarter wave-plates. The birefringent specimen is placed between the pairs in order to observe the photoelastic fringe patterns.

Polariser – an optical component which filters light so that the vectors for the light emitted are parallel to its axis.

Quarter wave-plate – an optical element which introduces in polarised light a uniform retardation of one quarter of the wavelength of the light.

Retardation – the phase shift produced between two co-linear light beams passing through a birefringent material.

5. Principles of the method

The phenomenon of photoelasticity is traditionally explained using the wave theory of light and this approach is adopted here. When a wave of plane-polarised light enters a transparent material subject to plane strain, it is divided into two component waves whose planes of vibration (directions of polarisation) coincide with the principal directions (direction of zero shear strain). The speed of travel of the two waves is proportional to the principal strain in their plane of polarisation. On leaving the material, one wave will be retarded relative to the other by an amount proportional to the difference in the principal strains and the length of the light path through the material. The relative retardation generates a phase difference which can be made visible by interference of the two co-linear waves. In photoelasticity, the interference is produced by a polariser that the resolves the two waves onto its polarising direction, which is usually perpendicular to the initial direction of polarisation of the light wave. The two waves interfere such that when the relative retardation is $N = 0, 1, 2, 3\ldots$ multiples of the wavelength of the light, destructive interference is produced and the material appears black, and when the retardation is $N = ½, 1½, 2½, 3½\ldots$ multiples of wavelength of the light, constructive interference is produced and a maximum light intensity is observed. When the difference in principal strain varies in the plane of the material a set of interference fringes are generated. These fringes are known as isochromatic fringes and can be assigned fringe orders, N. At locations in the material where the direction of the principal planes coincide with that of the polarisation in the polariser then the interference pattern is not generated and the material appears black. The loci of such points are known as isoclinics and can be used to evaluate the strain trajectories in the material.

The relationship of isochromatic fringes with the wavelength of light implies that in white light the fringes are coloured and exhibit the spectrum of the light employed. The centre of such fringes is considered to be the magenta/black between successive spectra, which is known as the tint-of-passage. In white light, isoclinics remain black. This facilitates the interpretation of the two types of fringe, alternatively a circular polariscope can be used to optically remove the isoclinics by placing a quarter wave-plate on both sides of the material which causes circularly polarised light to be employed rather than the plane polarised wave described above. More detailed explanations of the phenomenon can be found elsewhere [1-3].

Photoelasticity can be applied in a purely qualitative manner, or quantitatively using manual observation to obtain quantitative data at single points, or using digital methods to obtain full-field maps of strain magnitude and direction.

Reflection photoelasticity allows the strains in real components to be measured using a birefringent coating as a strain witness. A thin, transparent polymer coating of uniform thickness is bonded to the surface of the component under investigation. The coating either has a reflective layer at the interface with the adhesive or reflective particles are included in the adhesive layer. Subsequent strains induced in the surface of the component are reproduced in the coating and generate photoelastic fringes which may be analysed using a polariscope as described above but with the optical axis bent through 180° at the reflective layer. Polariscopes specifically designed for use with these coatings are available.

The application of photoelastic coatings is crucial to the successful application of the technique. The guidance below is based on and contains extracts from the technical notes [4-6] produced by Vishay Measurements Group Inc. and are reproduced with permission.

6. Apparatus

A polariscope consists of the following elements described from the light source to the sensor or viewing location: a light source which may be a white or a monochromatic source capable of generating a brightness of polarised field illuminating the sample of 300cd/m^2 [7]; a narrow band-pass filter to create monochromatic light of bandwidth no greater than 20nm [8]; a diffuser may also be included; a polariser element capable of producing a degree of polarisation of the field at all points not less than 99% [7]; a sensitive tint plate or quarter wave-plate, having a nominal optical retardation of 565nm and with its slow axis at 45° to the plane of polarisation; the specimen or sample of interest; a second quarter wave-plate with its slow axis perpendicular to the first one; and a second polariser with its axis of polarisation perpendicular to the first so that a dark field circular polariscope is generated. This arrangement should produce a magenta/black field of view, and is suitable for viewing isochromatic fringes. To observe the isoclinic fringes, the quarter wave-plates can be either physically removed or made optically inactive by rotating them to align their slow axes with those of the polarisers. The polarisers and quarter wave-plates should be located in mounts that allow rotation through known or measurable angles.

For a description of the setting up of a polariscope see section 3 of ASTM F218-95 [9]. The measurement of fringe orders up to 4, i.e. relative retardations of up to four times the wavelength of the light, is described in section 4 of ASTM F218-95. When used for quantitative measurements, particularly in the glass industry, polariscopes are sometimes described as polarimeters. Procedures for verifications and calibration of both individual elements and complete polarimeters can be found in ASTM C1426-99 [10].

7. Sample preparation

7.1 Selection of Coating [4]

The proper selection of photoelastic coating materials is as important to this method of experimental stress analysis as gauge and adhesive selection are to the strain gauge user. Although the selection of a coating material is largely a matter of common sense, it is helpful to follow a systematic procedure in order to avoid the omission of one or more important considerations. Naturally, the primary desire is to select a coating material that will give maximum reliability and accuracy under a given set of test circumstances, and do so with minimum effort and expense. Since there are numerous factors that affect the performance of a photoelastic coating, and a variety of performance requirements that are sometimes in conflict, a compromise is often necessary. The terms of the compromise are usually dictated by the ultimate purpose and conditions of the test.

The best practice is to list all the factors important to the particular application and satisfy the more critical requirements first. The following are the principal considerations in the selection of a photoelastic coating for a specified set of test conditions: method of plastic application to the test surface; sensitivity; contour severity; reinforcing effect; maximum elongation; and test temperature.

Photoelastic coatings are available in two basic forms: solid flat sheets and liquids for casting sheets that can be contoured. There are several different types of coating material available in each of the forms and these can be classified generally into three categories according to their elastic moduli; that is, high-, medium-, and low-modulus materials [11]. When the surface of the test part to be coated is flat, it is preferable to use flat sheets, since these offer the following advantages: uniform thickness (tolerance: ±0.05 to ±0.08 mm, depending upon the material type); uniform physical and photoelastic properties; and ease of handling. For irregularly shaped structures which cannot be coated with flat sheets, liquid plastic must be selected and applied using the contoured-sheet method [12]

7.2 Sensitivity of coating

Perhaps the single most important factor to be considered in the selection of a photoelastic coating is the birefringent sensitivity of the plastic material, since this property is involved in the basic equation used for photoelastic coating analysis:

$$\varepsilon_1 - \varepsilon_2 = N \cdot \frac{\lambda}{2t_c K} = N \cdot f \tag{1}$$

where ε_1, ε_2 are the principal strains, N is the fringe order or number of wavelengths of retardation, λ is the wavelength of the tint of passage in white light which is usually taken as 575nm, t_c is the coating thickness, K is the strain-optic coefficient of the photoelastic plastic and f is the fringe value of the plastic coating.

As a preliminary to conducting an experiment, the number of fringes required to be observed can be decided. Typically one to four is appropriate for manual analysis, and if the expected strain level can be estimated, then the desired coating sensitivity, or fringe value, can be calculated as follows:

$$f = \frac{\varepsilon_1 - \varepsilon_2}{N} = \frac{(anticipated\,strain)}{(desired\,N_{\max})} \tag{2}$$

Ideally, the expected strain level will correspond to incipient yielding of the material under stress analysis. In practice, however, a lower strain level is often imposed by specified test and loading conditions. Once the fringe value has been established from the expected strain level and number of fringes, the type and thickness of the plastic which will satisfy the sensitivity requirement can be determined with the following relationship:

$$f = \frac{\lambda}{2t_c K} \tag{3}$$

For convenience in plastic selection, this relationship has been plotted figure 1 [4]. The figure shows coating thickness and the ranges of the strain-optic coefficient for which there are available materials. To use the graph, enter along the ordinate at the appropriate value of f and project horizontally until an intersection with a sloping thickness line which falls within one of the crosshatched zones is found. This intersection defines a value of K, read from the abscissa, and a coating thickness that are consistent with the sensitivity requirement. If no such intersection can be found, it will generally be necessary to accept a lower sensitivity and work with fewer fringes.

7.3 Contour severity

Another instance when a thinner and less sensitive coating may be required occurs when sheets must be contoured over highly convoluted surfaces. If the surface to be coated has small-radius compound curvatures, it will be necessary to select a coating thickness such that the sheet can be contoured over the projections and into the recesses while maintaining uniform sheet thickness. As a general rule of thumb, the sheet thickness should be less than 20 percent of the radius of curvature of the surface. Somewhat greater thicknesses are satisfactory for simply curved surfaces.

7.4 Reinforcing effect

As noted earlier, there are certain cases in which a thick coating may produce a significant reinforcing effect that must be taken into account if accurate results are to be obtained. On structural members such as "I", "H", "U", or box beams and on heavy wall sections, tubular structures, castings and the like, the reinforcement caused by the plastic coating is negligible and can be ignored. The reinforcing effect is usually negligible for plane-stress problems (pressure vessels, plates, and panels with the load applied in the plane of the panel), and for membrane stresses produced with little or no bending. However, when thin beams or plates are subjected to bending, the plastic coating reinforces the test part noticeably, and the measured strain must be corrected for this effect. Also, in the case of low-modulus materials like plastics, the reinforcing effect for plane stress cannot be ignored, and must be corrected for. The factors responsible for the reinforcement error in bending are as follows:

1. The neutral plane shifts toward the coating.
2. The section is stiffened, and thus the curvature produced by a prescribed bending moment is smaller.
3. The photoelastic reading is averaged through the plastic.
4. The average strain in the coating is greater than the strain at the surface of the test specimen.

It will be noticed that the third and fourth factors above are not reinforcement effects as such, but photoelastic and geometric effects, respectively. However, all four effects act in concert, and it is convenient to lump the errors, thus permitting adjustment of the data with a single correction factor.

The correction factor, C, is a function of the ratio of the elastic moduli and of the thicknesses as between the coating and the specimen, and can be calculated analytically, as shown in sections 9.2 and 9.3. The correction factors C_{PS} and C_B represent the ratio of the actual strain to the measured strain at the surface of the test specimen. To obtain the actual strain, the measured strain is multiplied by the correction factor:

$$\textit{actual strain} = \textit{measured strain} \times C \tag{4}$$

It can be seen from the graph, that with very thin plastic coatings both C_{PS} and C_B approach unity, and no correction is necessary. In the lower central area of the graph, where the curves reverse directions, a large variation in the coating-to-specimen thickness ratio has relatively little effect on the magnitude of C_B. Also in this area, the sensitivity of the system is increased by the factor $1/C_B$, reflecting the fact that the strain in the coating, is higher than in the specimen. This characteristic is especially useful in the stress analysis of very thin plates in pure bending. With increasingly thicker coatings there is a point for each metal where C_B again becomes unity, eliminating the need for correction.

7.5 Maximum elongation

The maximum measurable strain for a particular photoelastic coating depends upon its stress-strain curve and the linearity of photoelastic behaviour. Coatings available commercially range in maximum elongation from 3% to 100%. The performance required of a coating for measuring fully plastic strains in metals is different from that for the elastic or elastoplastic ranges. With plastic strains, coating sensitivity is less significant because of the high strains present. The most critical consideration is the ability of the coating and adhesive to follow the metal into the plastic region. Using either a very thin coating of the plastics with higher elastic moduli, or a thicker coating of the lower elastic modulus plastics can solve this problem. The choice between these alternatives depends upon the information being sought on a particular application. For example, to investigate localised plastic deformation (Luder's lines) a thin coating of the high elastic modulus plastic would be preferred to minimise the reinforcement effect; but to analyse the stress distribution in the plastic range a thin coating with a ten percent elongation capability would be better.

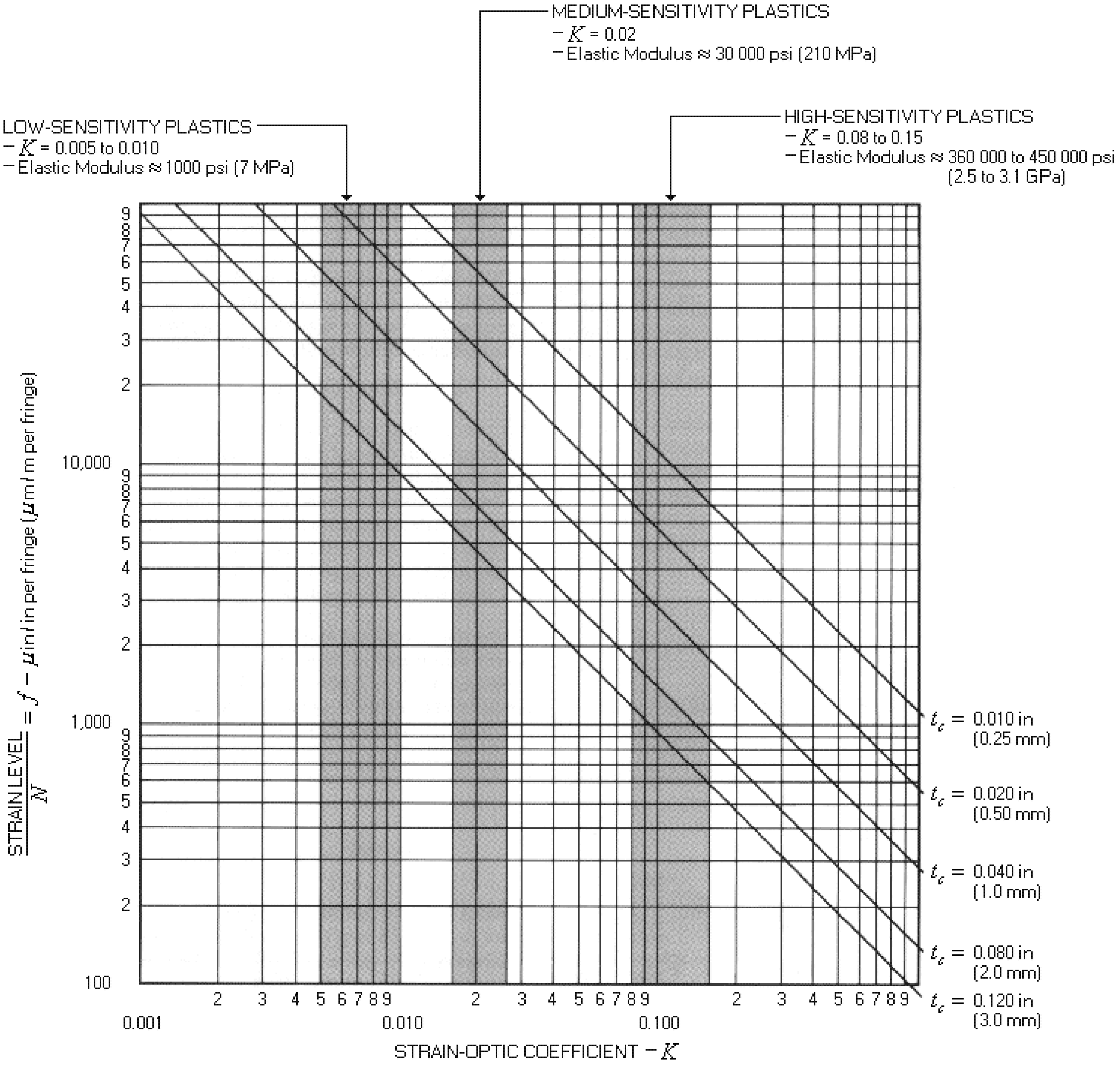

Figure 1: Parametric representation of expression (3) showing relationship between Strain-optic coefficient, fringe constant and coating thickness (reproduced from [4]).

7.6 Test temperature

If the test is to be performed at other than room temperature, consideration must be given to the effects of temperature on the behaviour of the coating. The coatings normally exhibit two temperature ranges over which *K* varies rapidly with temperature. While the coatings can be used in either region of constant *K*, it is important to select a material for which *K* remains constant throughout the entire temperature range of the test. There are additional thermal effects which must also be considered whenever tests are conducted at other than room temperature [13].

8. Calibration procedure [5]

In order to translate measured fringe orders in a photoelastic coating into strains or stresses in the coated test object, it is always necessary to introduce the strain-optic sensitivity of the coating. In reflection photoelasticity, the basic relationship between strain and fringe order is given in equation (1).

It is important to note the distinction between the coefficients *K* and *f*. The strain-optic coefficient *K* defines a fundamental property of the photoelastic material, and is independent of the plastic thickness or the length of the light path. The fringe value, *f* specifies the strain-optic sensitivity of a particular photoelastic coating (i.e., the difference in principal strains which will produce one fringe in that coating). As shown by equation (1), the fringe value accounts for the thickness of the coating, the fact that the light ray traverses the coating twice in reflection photoelasticity, and, finally, the nature of the light source. For typical photoelastic plastics used in the stress analysis of structural materials, *K* varies from 0.08 to about 0.15, with the larger coefficients corresponding to the more optically sensitive materials. The fringe value *f* can be adjusted (by selection of the coating thickness) to suit the stress analysis problem; but, for most practical cases, will fall in the range of 500 to 3000 micro strain per fringe, with the low fringe values representing the more sensitive coatings. The strain optic coefficient is usually given by the manufacturer, however for greater accuracy, a specimen from each sheet of photoelastic plastic should be calibrated for strain-optic sensitivity.

The recommended method of calibration is a cantilever beam with the coating to be calibrated bonded to its upper surface. The beam should be rigidly clamped at one end and loaded at its free end by a precision micrometer, permitting accurate measurement of the deflection. When a beam, to which a specimen of photoelastic coating has been bonded, is deflected by a predetermined amount, a known state of strain is imposed upon the coating. Measurement of the resultant birefringence in the coating provides the necessary information for relating fringe order to principal strain difference.

It is recommended that the beam should be manufactured from aluminium alloy (e.g. 2024-T4 or 7075-T6) bar with the following dimensions 6.35 ±0.025 x 25 x 318 mm. The thickness of the beam should be verified before the surface is cleaned and decreased. The following protocol can be followed to prepare the calibration test:

1. Cut a calibration specimen 25 x 76 mm from the sheet of photoelastic coating. In the case of contoured coatings, this is done with scissors, immediately after removing the sheet from the casting plate, while the plastic is still soft and suitable for shaping to contours. The calibration specimen should be placed back on a casting plate and allowed to polymerise completely in the form of a flat strip. When working with precast rigid sheets, the calibration specimen can be removed from the sheet with a jigsaw.
2. Before bonding the specimen to the calibration beam, measure and record the coating thickness, then clean and degrease the specimen thoroughly.

3. Mix a small batch of adhesive (the same used to bond the parent plastic to the test part), and apply a thin layer of the adhesive to the calibration beam where the coating is to be bonded. The calibration specimen should be located on the beam as specified above.
4. Apply the calibration specimen to the beam surface by first placing one end in contact with the adhesive (holding the other end up at a small angle), and then pressing down progressively along the length of the strip, squeezing out the excess adhesive in the process.
5. Finish the installation by building a fillet of adhesive at each end of the strip. Allow the adhesive to cure at room temperature for the time specified in the instructions for that adhesive.

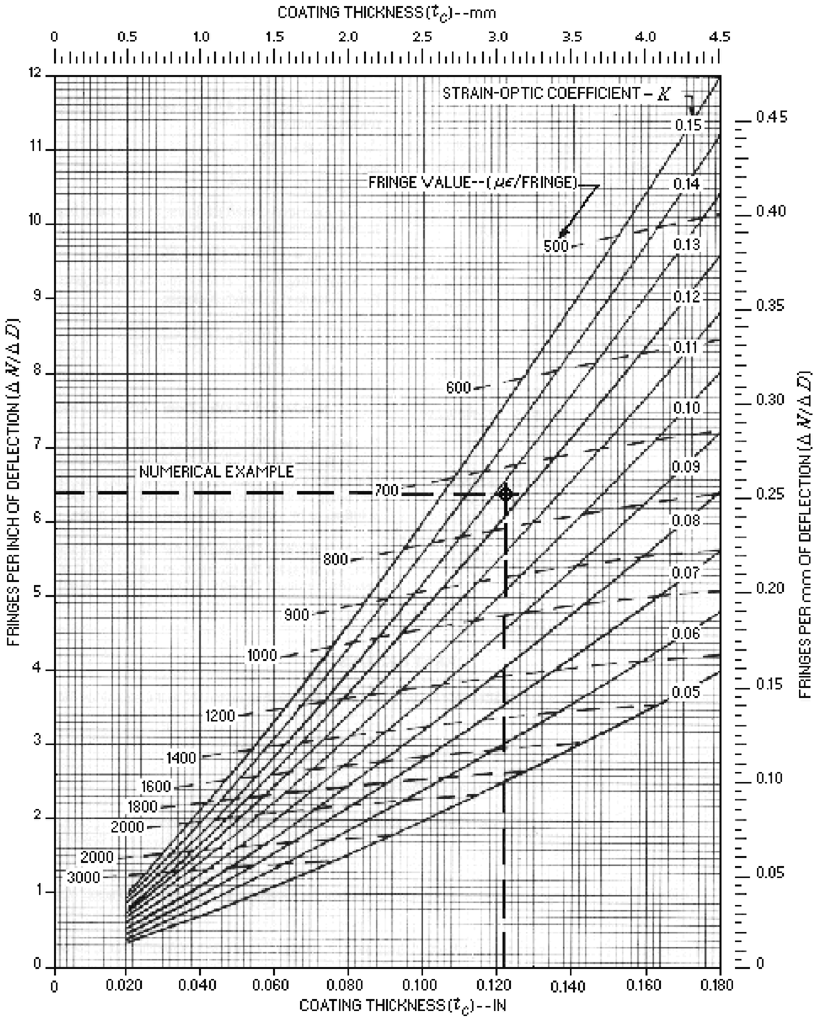

Figure 2: Calibration chart for photoelastic coatings (reproduced from [5]).

To make calibration measurements mount the beam with the calibration specimen adhered as a cantilever with the micrometer fixed to measure the deflection of the free end. Using a fine-pointed grease pencil, marking pen, or scriber, mark the coating with a small cross + on the centreline of the beam. Arrange the calibration beam and polariscope for normal incidence viewing and observe the calibration point while slowly rotating the micrometer head. When the spindle of the micrometer contacts the beam, slight birefringence will start to appear in the coating.

Continue rotating until the micrometer reading reaches a convenient round number (say, 0.001mm or 0.002 mm).

Accurately measure the fringe order at the pre-marked calibration point for the initial micrometer setting and record the result. Rotate the micrometer to achieve a suitable increment of deflection (e.g. 0.04mm), and make a new fringe-order measurement. Repeat the operation, making a measurement after every 0.04mm increment of deflection, and continuing for at least five increments

When calibrating a very thin coating, a total deflection greater than 13mm may be required to obtain readily measured fringe orders. However, caution should be exercised in the use of larger deflections to avoid exceeding the yield strength of the beam.

The resulting fringe order data should be plotted as a function of beam deflection, and a least-squares regression line fitted to the data in order to obtain $\Delta N/\Delta D$. The plotted points should fall on or very close to the line; and, if not, the measurements should be repeated with greater care, making certain that the beam is tightly clamped and centred. The value of $\Delta N/\Delta D$ can be used to enter the calibration chart in figure 2 and obtain values for the strain-optic coefficient K and/or the fringe value f directly from the graph, interpolating in each case as necessary. The calibration graph is based upon dimensions of the beam and calibration strip specified above, and includes all corrections for deflection-controlled bending. It is assumed in the development of the calibration graph that the adhesive had a representative adhesive thickness of 0.075 mm. If the adhesive is noticeably thicker or thinner than this, and if the effort is warranted by the accuracy requirements, the coefficient K can be corrected with the following relationship:

$$K = K'\left[1 + \frac{2(t_A - t_B)}{t_B + t_C}\right] \tag{5}$$

where K' is the strain-optic coefficient from the calibration graph for the measured data, K is the actual or corrected strain-optic coefficient, t_A is the actual adhesive thickness, t_B the beam thickness and t_C the coating thickness.

To determine the actual adhesive thickness, measure the overall thickness of the coated beam with a micrometer and subtract away the pre-measured thicknesses of the bare aluminium beam and the calibration strip.

9. Recording and measurement procedures

The birefringence parameters can be measured using either goniometric or compensation techniques. Compensation techniques involve the use of a physical compensator with a known and variable birefringence. These devices should be used following the procedures described in ASTM D4093-95 [14]. The following procedure should be used for goniometric measurements using a polarimeter set-up as described in ASTM F218-95 [6]. With the quarter wave-plates removed or rendered optically inactive by aligning their optical slow axis with the axes of the polariser and analyser, rotate all the elements of the polarimeter until an isoclinic fringe completely covers the point of interest. The angle of rotation required to achieve extinction by the isoclinic fringe is the angle of the principal strains to the polarising axes of the polarimeter. Insert the quarter wave-plates with their slow axes at 45° to the polarising axes of the polariser and analyser as in the original configuration. Now, rotate the analyser only in order to move an isochromatic fringe over the point of interest. The angle of rotation, β (in degrees) of the analyser required to achieve this second extinction indicates the fraction of a fringe present at the point, such the total fringe order present is given by:

$$N = n \pm \beta/\pi \tag{6}$$

where n is the fringe order moved to the point of interest by the rotation of the analyser. When a higher order fringe is moved down the fringe gradient the fraction β/π is subtracted from the fringe order, *n*, and vice versa for fringe orders moved up the fringe gradient.

9.1 Fringe-Order Corrections [10]

All instrumentation systems have in common two intrinsic characteristics which inevitably limit their accuracy in varying ways and degrees: first, the tendency to respond to other variables in the environment in addition to the variable under investigation, and second, the tendency to alter the variable being measured. The user of the reflection polariscope should keep in mind the fact that the photoelastic coating method is not an exception to the foregoing generalities. There are two generally applicable approaches to minimizing the ever-present instrumentation errors. The first of these involves the careful design, planning, and execution of the measurement by a knowledgeable test engineer so as to compensate for, and/or control, those error sources which are subject to such treatment. Following this, the test engineer must employ the second defence against errors, which is to apply corrections to the measurement as appropriate. Both of these approaches can and should be utilised in the process of strain measurement with photoelastic coatings. The principal sources of error in the photoelastic coating method are as follows: parasitic (initial) birefringence; reinforcement effects in plane-stress systems; reinforcement and strain-extrapolation effects for plates in bending; and temperature effects.

Any initial colour pattern in a photoelastic coating (prior to applying test loads) causes an error in subsequent fringe-order measurements, which must be corrected. Under normal circumstances, residual birefringence in the coating is produced only by severe mishandling of the plastic during, or after application to the test object. In such cases, it is usually preferable to strip off the coating and apply a new one, rather than attempting to make corrections for the parasitic birefringence. There are instances, however, when the nature of the test circumstances may unavoidably introduce residual birefringence, and necessitate correction of the fringe-order measurements.

Following are several examples:

A. *Residual birefringence caused by a difference between the temperature at which the coating was bonded in place and the test temperature.*

This parasitic birefringence is produced by differential thermal expansion between the coating and the test object. The initial birefringence is concentrated at free edges and decreases with distance from the edge. For homogeneous, isotropic test materials, it approaches zero at distances greater than four times the coating thickness. At points far removed from the edges, the stress state in the plane of the coating due to differential thermal expansion is equal biaxial stress (σ_1-σ_2), and produces no birefringence.

B *Parasitic birefringence due to contraction of the cement.*

Over periods of a month or so, the cement used to bond the coating to the test part may continue to polymerise and, in so doing, contract. The effect is similar to that in Item A, and is concentrated at the edges.

C *Edges not protected against humidity.*

If the edges of the plastic coating are not protected from humidity by a layer of cement, some moisture may be absorbed through the finished edges of the plastic. The result will be swelling of the plastic along the edges, producing parasitic birefringence in these areas.

In all of the foregoing cases, the residual birefringence is localised near the edges; and, if the edge of the coating matches the edge of the test object, a very simple correction procedure can be applied. At every point on the free or unloaded boundary of a test object, the principal axes are tangential and perpendicular to the boundary and the boundary itself is an isostatic, or principal stress trajectory. This is equally true of the stress at the edge of the coating, whether caused by test loads or by the effects described above. Because the load-induced and parasitic birefringence are congruent, direct superposition can be employed, and correction can be made at all points on the free boundary by simply subtracting the measured fringe order under no load from that measured with the test loads applied. It is important to note that direct linear superposition of stress states is permissible only when the directions of the principal stresses coincide for the two states of stress.

When residual birefringence exists in the coating due to mishandling of the plastic, or to yielding of the test part after it has been coated, the directions of the principal stresses causing the parasitic birefringence will not generally coincide with the principal axes produced by the test loading. In such cases, the correction cannot be made by simple subtraction, and other methods must be employed. Before proceeding, it is necessary to test whether the directions of the principal axes associated with the parasitic birefringence coincide with those under loading. For this purpose, first trace or closely observe the isoclinic patterns of the parasitic birefringence with no load applied to the test part. Then load the part, and examine the isoclinics again. If the isoclinic patterns under load and no-load conditions are identical i.e., the isoclinics do not move as load is applied, both states of stress have the same principal axes, and the parasitic birefringence can be subtracted algebraically from that "caused by" or "introduced with" load. If the isoclinics move with the application of load, then the subtraction must be performed using vector subtraction:

$$N = \sqrt{N_f^2 + N_i^2 - 2N_f N_i \cos 2\left(\theta_f - \theta_i\right)} \tag{7}$$

where N is the actual fringe order due to the applied load, N_i is the initial parasitic birefringence, N_f is the final birefringence measured under the applied load, θ_i is the angle between the reference axis and the direction of the major principal stress associated with the initial birefringence which can be deduced from the isoclinic angle and θ_f is the corresponding angle for the final birefringence.

The angle between the horizontal reference axis and the major principal stress induced by the applied load is:

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{N_f \sin 2\theta_f - N_i \sin 2\theta_i}{N_f \cos 2\theta_f - N_i \cos 2\theta}\right) \tag{8}$$

The suggested procedure to correct for parasitic birefringence is as follows:

1. With no load on the test object, measure the fringe order, N_i, of the initial parasitic birefringence at the test point, and the isoclinic parameter, θ_i, in order to deduce the angle of the major principal stress.
2. After application of the test load, again measure the fringe order, N_f and the isoclinic angle θ_f. Note that these measurements result from the combination of parasitic and load-induced birefringence.
3. Calculate the corrected fringe order, N, resulting only from the test load, from expression (7)
4. Calculate the isoclinic angle, θ, between the reference direction and the axis of the load-induced major principal stress from expression (8).

9.2 Reinforcement Effects in Plane-Stress Problems

The term "plane stress" is generally applied to structural members such as plates and panels which are loaded only in their mid-planes, and are not subject to significant out-of-plane bending moments. Thin-walled pressure vessels and certain other structures can also be treated approximately as plane-stress problems.

When a plane-stress structural member to which a photoelastic coating has been bonded is subjected to loads, the coating reinforces the member and carries part of the load. As a result, the strains in the test member are lower than they would be without the coating present. The reinforcement error is very small for metal structures, and can often be ignored. However, when the test object is made from plastics or other non-metals, the error is generally significant, and a correction is required.

The correction relationship for reinforcement effects in plane-stress situations can be expressed as[15]:

$$C_{PS} = 1 + \frac{E_c}{E_s}\frac{t_c}{t_s} \tag{9}$$

where C_{PS} is the factor by which the observed fringe order in plane stress must be multiplied to obtain the corrected fringe order, E_c and E_s are the elastic moduli of the coating and specimen respectively and t_c and t_s are the coating and specimen thickness respectively.

When loading is applied by an in-plane fixed displacement, the strain field is the same for coated and uncoated test parts. Consequently, there is no plane-stress correction.

9.3 Reinforcement and Strain Extrapolation Effects in Bending

A. *Applied Bending Moments*

When thin beams, plates, or shells are subjected to bending moments, the effects of the photoelastic coating on the structural member are generally much greater than for the plane-stress case; and a correction is almost always required. The influence of the coating on a member in bending is quite complex, and the correction factor must account for three different effects as follows:

1. The neutral axis of the coated member is shifted toward the coated side.
2. The coating increases the stiffness of the member, and decreases the deformation (curvature) for a particular applied bending moment.
3. There is a strain (and fringe-order) gradient through the thickness of the coating. The polariscope measures the average fringe order at the mid-plane of the coating, which is further from the neutral axis than the surface of the test member.

The first two of the above effects tend to depress the observed fringe order compared to the correct value, and the third tends to exaggerate the fringe-order indication. All three effects operate simultaneously, but are influenced differently by the elastic-modulus and thickness ratios, $E_R = (E_c / E_s)$ and $t_R = (t_c / t_s)$. A single correction factor for all three effects is presented by Zandman et al [15]:

$$C_B = \frac{1 + E_R\left(4t_R + 6t_R^2 + 4t_R^3\right) + E_R^2 t_R^4}{1 + t_R} \tag{10}$$

where C_B is the factor by which the observed fringe order in bending must be multiplied to obtain the corrected fringe order. The strain-exaggeration effect is predominant with high-elastic-modulus materials, and for such materials the observed fringe order is usually too high, and must be multiplied by a factor less than unity. However, for low-elastic-modulus materials, the stiffening effect of the coating predominates, and the measured fringe order is commonly too low, requiring a correction factor greater than unity.

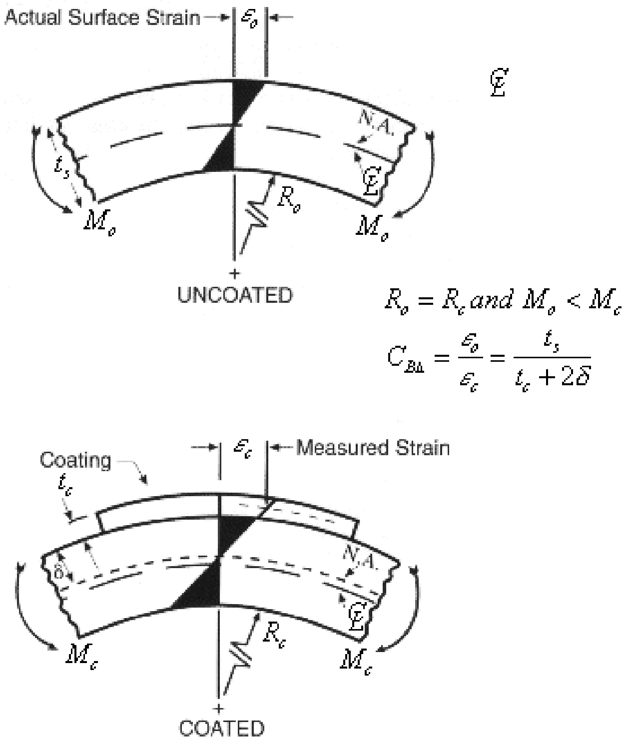

Figure 3: Schematic showing bending reinforcement (reproduced from [6]).

B. *Imposed Bending or Flexural Deformation*

The correction factor $C_{B\Delta}$ for imposed flexural deformation differs from the correction factor C_B for applied bending moment loading. Bending to a predetermined deformation requires discrete radii of curvatures in the deformed part. This is illustrated below where the imposed radii of curvature in the uncoated and coated parts are shown to be equal, $R_O = R_C$

Indeed, the flexural rigidity of the coated part is greater than the uncoated part and, consequently, a larger bending moment is required to deform the coated part, $M_C > M_O$. Simple geometric consideration of the imposed curvature provides:

$$C_{B\Delta} = \frac{1 + E_R t_R}{1 + t_R} \qquad (11)$$

It is clear that $C_{B\Delta}$ is relatively independent of the type of photoelastic coating applied to metal structures. However, $C_{B\Delta}$ is dependent on the properties of the photoelastic coating when the test objects are made from lower modulus plastics.

9.4 Correction for the Effects of Temperature Changes

Whenever the temperature changes during a test, a system of stresses will develop in the plastic coating as a result of the difference in thermal expansion coefficients between the structure and the plastic. Under certain circumstances, this condition can produce errors in the observed fringe orders, and necessitate corrective measures. Although a full treatment of thermal effects is given by Zandman et al. [13] the following procedures will be adequate for most practical applications.

A. *Regions Not Located on Boundaries*

At interior points, away from the edges, the effect of differential expansion between the coating and the structure is to produce a plane state of stress ($\sigma_1 = \sigma_2$) in the coating. Whether or not a correction must be introduced depends upon whether a normal- or oblique-incidence measurement is made. In this case interior points are defined as those at distances from the edges of the coating greater than four times the coating thickness.

Normal incidence: In normal incidence, a plane state of stress, for which the principal stresses in the coating are equal, causes no birefringence. Therefore, no correction is necessary, and the observed fringe orders are used directly to determine the mechanical and/or thermal stresses in the structure, assuming that it is fabricated from a homogeneous, isotropic material.

Oblique incidence: When the coating is observed in oblique incidence, the temperature-induced stress state produces a difference in the secondary principal stresses (in the plane perpendicular to the oblique light ray). The result is a superimposed birefringence (or "zero-shift") which is calculable from:

$$N_0(\Delta T) = \frac{1}{f}\left(\frac{1+\upsilon_c}{1-\upsilon_c}\right)\frac{\sin^2\theta}{\cos\theta}(\alpha_s - \alpha_c)\Delta T \tag{12}$$

where $N_0(\Delta T)$ is the birefringence due to temperature change only, irrespective of the state of stress in the structure; υ_c is the Poisson's ratio of the photoelastic coating; α_s and α_c are the thermal expansion coefficients of structure and coating, respectively; θ is the angle of oblique incidence from the surface normal; and ΔT is the change in temperature.

Since $N_0(\Delta T)$ is independent of direction in the plane of the coating, correction can be made by algebraic subtraction of the zero-shift from the observed fringe order. That is,

$$N_\theta = \hat{N}_\theta - N_0(\Delta T) \tag{13}$$

where N_θ is the corrected birefringence in oblique incidence and $\hat{N}_\theta$ is the observed or uncorrected birefringence including the temperature-induced zero-shift.

B. *Free Edges and Boundaries*

Because there is only one nonzero principal stress on any free edge or boundary, oblique-incidence measurements are not made there. In normal incidence, however, fringes will appear at the edges due to temperature change; and, for a particular temperature change, the sign and magnitude of the temperature-induced birefringence depend upon the sign and magnitude of the edge curvature.

The most effective procedure for this case is to employ a dummy specimen having the same configuration as the test part, coated with the same plastic in the same thickness, and subjected to the same thermal environment, but always left free of mechanical and thermal stresses. The measured birefringence on the dummy specimen at any temperature then represents the temperature-shifted zero for measurements on the actual test part. Because the direction of the only nonzero principal stress at a free boundary is always tangential to the boundary, irrespective of what caused the stress, the correction is made by direct subtraction, in the manner of expression (13).

The same dummy specimen can be used in the same manner to obtain the zero-shift for oblique-incidence measurements at interior points, away from the edges. Because the stress state induced at interior points by a temperature change is plane ($\sigma_1 = \sigma_2$) and has no directional properties, the direct correction can again be made with expression (13).

In those cases where the actual coated test part can be subjected to the test temperature while remaining free of thermal and mechanical stresses, no separate dummy is necessary. Zero readings can be made at the test temperature, either on the boundary in normal incidence, or at interior points in oblique incidence and the correction performed with expression (13) or its equivalent.

10. Data processing procedures

The majority of photoelastic data are collected manually on a point-by-point basis as a fringe order and isoclinic angle. It is simple calculation to apply the fringe constant and convert the data to principal strain difference using expression (1). Digital photoelasticity [16] is just appearing as a viable approach for recording reflection photoelastic data but is not in routine use for quantitative measurements. However, it does offer the potential to separate the principal strains by combining the data with that from thermoelasticity where the sum of the principal strains is obtained [17]. This new approach appears to be more robust than traditional methods [18] which are rarely used in reflection photoelasticity due the difficulty of application and the high level of noise usually obtained in the data.

11. Areas of applications

Reflection photoelasticity can be employed to determine the strains in a wide range of applications although until recently only very limited quantitative data was recorded, often at only a few points. One of the principal applications was to identify the locations for strain gauges. However, it has been used for the analysis of airframes [19], residual stresses [20] and in fracture mechanics [21].

12. Bibliography

1. Doyle, J.F., Phillips, J.W., *Manual on Experimental Stress Analysis,* Society for Experimental Mechanics, Bethal, CT, 1989.
2. Cloud, G., *Optical Methods of Engineering Analysis*, Cambridge University Press, Cambridge, UK,1995.
3. Frocht, M.M., *Photoelasticity*, John Wiley & Sons, New York, 1941.
4. TN-704: *How to select photoelastic coatings*. Vishay Measurements Group, Inc., Raleigh NC., 1978.
5. TN-701: *Calibration of photoelastic coatings*. Vishay Measurements Group, Inc., Raleigh NC., 1977.
6. TN-706-1: *Corrections to photoelastic fringe-order measurements*. Vishay Measurements Group, Inc., Raleigh NC., 1992.
7. C148-00: *Standard test methods for polariscopic examination of glass containers*. ASTM International, West Conshohocken, PA. 2000.
8. Ji, W., Patterson, E.A., "Simulation of errors in automated photoelasticity", *Experimental Mechanics*, 38 (2), 132-139, 1998.
9. F218-95: *Standard test method for analyzing stress in glass*. ASTM International, West Conshohocken, PA. 1995.
10. C1426-99: *Standard practices for verification of calibration of polarimeters*. ASTM International, West Conshohocken, PA. 1999.
11. Bulletin S-116: *Photoelastic Materials and Coatings* Vishay Measurements Group, Inc., Raleigh NC., 1978.
12. Bulletin IB-221: *Instructions for casting and contouring photoelastic sheets*. Vishay Measurements Group, Inc., Raleigh NC., 2001.
13. Zandman, F.S,. Redner A.S., Post D., "Photoelastic-coating Analysis in Thermal Fields", *Experimental Mechanics,* 3, 215-221, 1963.
14. D4093-95: *Standard test method for photoelastic measurements of birefringence and residual strains in transparent or translucent plastic materials*. ASTM International, West Conshohocken, PA. 1995.
15. Zandman, F.S. Redner A.S., Riegner E. I, "Reinforcing Effect of Birefringent Coatings", *Experimental Mechanics,* 2, 55-64 1962.
16. Patterson, E.A., 'Digital photoelasticity: principles, practice and potential', *Strain*, 38, 27-39, 2002.
17. Barone, S., Patterson, E.A., 'The development of simultaneous thermo- and photo-elasticity for principal stress analysis', *Strain*, 35 (2), 57-65, 1999.
18. Zandman, F., Redner, A.S., Dally, J.W., *Photoelastic coatings*, SESA Monographs No. 3, Iowa State University Press, Ames, 1977.
19. O'Brien, E.W., "Better than average stress model – photoelastic analysis for Airbus design", *Proc. IMechE. Part G, J. Aerospace Engineering*, 207 (2), 133-137, 1993.
20. Corby, T.W. Jr., Nickola, W.E., "Residual strain measurement using photoelastic coatings", *Optics & Lasers Engng.*, 27 (1), 111-123, 1997.
21. Nurse, A.D., Patterson, E.A., "Photoelastic measurement of stress intensity factors during a complete fatigue cycle", in *Recent Advances in Experimental Mechanics,* Silva Gomes et al (eds), Balkenna, Rotterdam, pp. 195-199, 1994.